UNIT A

A1: Combinatorics

Permutations:
- problems involving permutations
- use the multiplication principle and factorial notation
- permutations and restrictions with or without repeated objects

The inclusion-exclusion principle for the union of two sets and three sets:

Combinations
- problems involving combinations
- \( \binom{n}{r} \) or \( ^nC_r \)
- Pascal’s triangle

A2: Vectors in the plane

Representing vectors:
- magnitude and direction of a vector
- vectors - displacement and velocity
- scalar multiple of a vector
- triangle rule to find the sum and difference of two vectors.

Algebra of vectors:
- ordered pair notation and column vector notation to represent a vector
- unit vectors and the perpendicular unit vectors
- vector in component form
- addition and subtraction of vectors in component form
- multiplication by a scalar of a vector in component form
- scalar (dot) product
- parallel and perpendicular vectors
- projections of vectors
- problems involving displacement, force and velocity

A3: Geometry

The nature of proof:
- implication, converse, equivalence, negation, contrapositive
- proof by contradiction
• use the symbols for implication ($\Rightarrow$), equivalence ($\Leftrightarrow$), and equality ($=$)
• use the quantifiers ‘for all’ and ‘there exists’
• examples and counter-examples

Circle properties and their proofs:
• Angles in a semicircle / circle
• Chords
• Alternate segment theorem
• Secant and tangent to a circle
• Problems finding unknown angles and lengths

Geometric proofs using vectors in the plane:
• Diagonals of a parallelogram

• Midpoints of a quadrilateral
UNIT B

B1: Trigonometry

The basic trigonometric functions:

- solutions of sin, cos or tan
- graph functions of sin, cos, or tan
- compound angles
- angle sum, difference and double angle identities

The reciprocal trigonometric functions, secant, cosecant and cotangent and their graphs:

- graph and simple transformations

Trigonometric identities:

- Pythagorean identities
- Products of sines and cosines expressed as sums and differences
- solve trigonometric equations
- trigonometric identities

Applications of trigonometric functions:

- model periodic motion using sine and cosine functions
- relevance of the period and amplitude of these functions

B2: Matrices

Matrix arithmetic:

- matrix definition and notation
- addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity and inverse
- determinant and inverse of 2 × 2 matrices and solve matrix equations of the form AX = B

Transformations in the plane:

- translations and representation as column vectors
- basic linear transformations: dilations, rotations and reflection
- representations of the transformations by 2 × 2 matrices
- transformations to points in the plane and geometric objects
- composition of linear transformations and the corresponding matrix products
- inverses of linear transformations and the relationship with the matrix inverse
- relationship between the determinant and the effect of a linear transformation on area
- geometric results by matrix multiplications

Topic 3: Real and complex numbers

Proofs involving numbers:
• Rational and irrational numbers:
  • rational numbers as terminating or recurring decimals and vice versa
  • prove irrationality by contradiction

Proof by mathematical induction:
  • nature of inductive proof including the ‘initial statement’ and inductive step
  • problems related to sums and divisibility

Complex numbers:
  • imaginary number
  • complex numbers in the form a + bi
  • complex conjugates
  • complex-number arithmetic: addition, subtraction, multiplication and division.

The complex plane:
  • complex numbers as points in a plane, Cartesian coordinates
  • addition of complex numbers as vector addition in the complex plane
  • location of complex conjugates in the complex plane

Roots of equations:
  • real quadratic equations
  • complex conjugate solutions of real quadratic equations
  • linear factors of real quadratic polynomials.

UNIT C

C1: Complex numbers

Cartesian forms:
  • review real and imaginary parts \( \text{Re}(z) \text{ and } \text{Im}(z) \) of a complex number \( z \)
  • review Cartesian form
  • review complex arithmetic using Cartesian forms

Complex arithmetic using polar form:
  • prove basic identities involving modulus and argument
  • Cartesian and polar form conversion
  • multiplication, division, and powers of complex numbers in polar form and the geometric interpretation
  • De Moivre’s theorem for integral powers

The complex plane (the Argand plane):
• addition of complex numbers as vector addition in the complex plane
• multiplication as a linear transformation in the complex plane
• identify subsets of the complex plane

Roots of complex numbers

• $n^{\text{th}}$ roots of unity and their location on the unit circle
• $n^{\text{th}}$ roots of complex numbers and their location in the complex plane

Factorisation of polynomials:

• factor theorem and the remainder theorem for polynomials
• conjugate roots for polynomials with real coefficients
• solve simple polynomial equations.

C2: Functions and sketching graphs

Functions:

• composition of two functions
• one-to-one function
• consider inverses of one-to-one function
• reflection property of the graph and its inverse

Sketching graphs:

• absolute value for the real number and graph
• relationships between the graph
• sketch the graphs of simple rational functions

C3: Vectors in three dimensions

The algebra of vectors in three dimensions:

• concepts of three dimensional vectors
• geometric results in a plane and simple proofs in three-dimensions

Vector and Cartesian equations:

• Cartesian coordinates for three-dimensional space, including plotting points and the equations of spheres
• vector equations of curves in 2D and 3D involving a parameter, and determine a ‘corresponding’ Cartesian equation in the 2D
• vector equation of a straight line and straight-line segment, given the position of two points, or equivalent information, in both two and three dimensions
• position of two particles each described as a vector function of time, and determine if their paths cross or if the particles meet
• cross product to determine a vector normal to a given plane
• vector and Cartesian equations of a plane and of regions in a plane
Systems of linear equations:

- general form of a system of linear equations in several variables,
- elementary techniques of elimination to solve a system of linear equations
- three cases for solutions of systems of equations –
  - a unique solution
  - no solution, and
  - infinitely many solutions
- geometric interpretation of a solution of a system of equations with three variables

Vector calculus:

- consider position of vectors as a function of time
- Cartesian equation of a path given as a vector equation in two dimensions including ellipses and hyperbolas
- differentiate and integrate a vector function with respect to time
- equations of motion of a particle travelling in a straight line with both constant and variable acceleration
- vector calculus -- motion in a plane including projectile and circular motion
UNIT D

D1: Integration and applications of integration
Integration techniques:
• integrate using the trigonometric identities
• integrate expressions of the form \( f(g(x))g'(x) \)
• integrate \( \frac{1}{x} \) \( dx = \ln |x| + c \), for \( x \neq 0 \)
• integrate \( \int \frac{1}{\sqrt{a^2 - x^2}} \) and \( \int \frac{a}{a^2 + x^2} \)
• partial fractions where necessary for integration in simple cases
• integrate by parts

Applications of integral calculus:
• areas between curves determined by functions
• determine volumes of solids of revolution about either axis
• numerical integration using technology
• probability density function, \( f(t) = \lambda e^{-\lambda t} \) for \( t \geq 0 \), of the exponential random variable with parameter \( \lambda > 0 \), and
• exponential random variables and associated probabilities and quantiles to model data and solve practical problems

Topic 2: Rates of change and differential equations
• implicit differentiation to determine the gradient of curves
• Related rates as instances of the chain rule
• solve simple first-order differential equations
• examine slope (direction or gradient) fields of a first order differential equation
• formulate differential equations including the logistic equation e.g. chemistry, biology and economics, in situations where rates are involved

Modelling motion:
• momentum, force, resultant force, action and reaction
• constant and non-constant force
• understand motion of a body under concurrent forces
• solve problems involving motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion and acceleration

Topic 3: Statistical inference
Sample means:
• examine the concept of the sample mean $\bar{X}$ as a random variable whose value varies between samples where $X$ is a random variable with mean $\mu$ and the standard deviation $\sigma$

• random sampling, distribution of $\bar{X}$ across samples of a fixed size $n$, including its mean $\mu$, its standard deviation $\sigma/\sqrt{n}$ (where $\mu$ and $\sigma$ are the mean and standard deviation of $X$), and its approximate normality if $n$ is large

• approximate standard normality of large samples ($n \geq 30$), where $s$ is the sample standard deviation.

Confidence intervals for means:

• concept of an interval estimate for a parameter associated with a random variable

• confidence interval, as an interval estimate for $\mu$, the population mean, where $z$ is the appropriate quantile for the standard normal distribution

• illustrate variations in confidence intervals between samples

• use $\bar{x}$ and $s$ to estimate $\mu$ and $\sigma$,

• collect data and construct an approximate confidence interval

• estimate a mean and report on survey procedures and data quality

References:

1. For full details about any ACT curriculum, please refer to ACT Board of Senior Secondary Studies (BSSS Courses); http://www.bsss.act.edu.au/curriculum/courses