### *Proof*

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}+C\right)=\frac{(n+1)x^{n+1-1}}{n+1}$$
$$=x^n$$

## EXAMPLES

**1.** The gradient of a curve is given by $\frac{dy}{dx}=6x^2+8x$. If the curve passes through the point $(1,-3)$, find the equation of the curve.

### *Solution*

$$\frac{dy}{dx}=6x^2+8x$$
$$y=6\left(\frac{x^3}{3}\right)+8\left(\frac{x^2}{2}\right)+C$$
$$\therefore\quad y=2x^3+4x^2+C$$

The curve passes through $(1,-3)$

$$\therefore\ -3=2(1)^3+4(1)^2+C$$
$$=2+4+C$$
$$-9=C$$

Equation is $y=2x^3+4x^2-9$.

**2.** If $f''(x)=6x+2$ and $f'(1)=f(-2)=0$, find $f(3)$.

### *Solution*

$$f''(x)=6x+2$$
$$f'(x)=6\left(\frac{x^2}{2}\right)+2x+C$$
$$=3x^2+2x+C$$

Now $f'(1)=0$

So $0=3(1)^2+2(1)+C$

$-5=C$

$$\therefore\quad f'(x)=3x^2+2x-5$$
$$f(x)=3\left(\frac{x^3}{3}\right)+2\left(\frac{x^2}{2}\right)-5x+C$$
$$=x^3+x^2-5x+C$$

CONTINUED

Now $f(-2) = 0$

So $0 = (-2)^3 + (-2)^2 - 5(-2) + C$

$= -8 + 4 + 10 + C$

$-6 = C$

$\therefore \quad f(x) = x^3 + x^2 - 5x - 6$

$f(3) = 3^3 + 3^2 - 5(3) - 6$

$= 27 + 9 - 15 - 6$

$= 15$

## 2.12 Exercises

**1.** Find the primitive function of
(a) $2x - 3$
(b) $x^2 + 8x + 1$
(c) $x^5 - 4x^3$
(d) $(x - 1)^2$
(e) $6$

**2.** Find $f(x)$ if
(a) $f'(x) = 6x^2 - x$
(b) $f'(x) = x^4 - 3x^2 + 7$
(c) $f'(x) = x - 2$
(d) $f'(x) = (x + 1)(x - 3)$
(e) $f'(x) = x^{\frac{1}{2}}$

**3.** Express $y$ in terms of $x$ if
(a) $\frac{dy}{dx} = 5x^4 - 9$
(b) $\frac{dy}{dx} = x^{-4} - 2x^{-2}$
(c) $\frac{dy}{dx} = \frac{x^3}{5} - x^2$
(d) $\frac{dy}{dx} = \frac{2}{x^2}$
(e) $\frac{dy}{dx} = x^3 - \frac{2x}{3} + 1$

**4.** Find the primitive function of
(a) $\sqrt{x}$
(b) $x^{-3}$
(c) $\frac{1}{x^8}$
(d) $x^{-\frac{1}{2}} + 2x^{-\frac{2}{3}}$
(e) $x^{-7} - 2x^{-2}$

**5.** If $\frac{dy}{dx} = x^3 - 3x^2 + 5$ and $y = 4$ when $x = 1$, find an equation for $y$ in terms of $x$.

**6.** If $f'(x) = 4x - 7$ and $f(2) = 5$, find $f(x)$.

**7.** Given $f'(x) = 3x^2 + 4x - 2$ and $f(-3) = 4$, find $f(1)$.

**8.** Given that the gradient of the tangent to a curve is given by $\frac{dy}{dx} = 2 - 6x$ and the curve passes through $(-2, 3)$, find the equation of the curve.

**9.** If $\frac{dx}{dt} = (t - 3)^2$ and $x = 7$ when $t = 0$, find $x$ when $t = 4$.